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THE MAGNITUDE OF THE PULFRICH STEREOPHENOMENON AS A FUNCTION OF DISTANCE OF OBSERVATION*

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The present experiment is concerned with a stereoscopic effect first described and analyzed by Pulfrich *(2)*. The stereophenomenon is readily demonstrated by means of a pendulum-bob made to oscillate in a frontal plane and on a level with the eyes. With binocular fixation maintained at a point directly below the oscillating bob, midway between the end-points of its swing, the path traversed by the bob appears to be confined approximately to the plane of oscillation and identical for either direction of stroke. When a smoked glass is placed before one of the eyes, however, the oscillating bob appears to rotate out of its plane of oscillation. The direction of the apparent rotation of the bob reverses itself when the smoked glass is placed before the other eye. As viewed from above, the apparent rotation is clockwise when the smoked glass decreases the illumination of the left eye and counterclockwise when it decreases the illumination of the right eye. The effect becomes noticeable at some threshold value and increases as the difference in the intensity of illumination between the two eyes increases. The effect is also elicited when a colored filter is substituted for the smoked glass.

THEORETICAL EXPLANATION

As analyzed by Pulfrich, the phenomenon is shown to be consistent with the law of corresponding retinal points and the general principles of space perception. The explanation is based on an hypothesized visual latent period whose magnitude is presumed related to a reciprocal function of the intensity of the illumination.

Simultaneous stimulation of corresponding retinal points will result in simultaneous "excitations," but only after a brief delay which becomes shorter as the stimulus-illumination increases. By placing the smoked glass before one eye, the stimulus-intensity for that eye is decreased. As a result, the time course of "excitations" from corresponding retinal points will no longer be simultaneous for the two eyes. Hence, for a

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transversely moving object, synchronous "excitations," at any given moment, will have arrived from slightly different retinal points, the amount of disparity increasing as the difference in brightness existing between the two retinal images becomes greater. Accordingly, the stereoscopic illusion produced is that the oscillating bob appears to move in a path which locates the bob nearer than it really is for the one direction of stroke, and farther than it really is for the return stroke.

GEOMETRIC ANALYSIS

An understanding of the effect may be obtained from an analysis of Figure 1 which is drawn to represent the horizontal plane of fixation, and contains the centers of rotation of the two eyes designated by Z_R and Z_L. The figure is meant to represent the stereoscopic space-image at the exact moment that the bob, P_{LR}, moving from left to right in its path W_1W_2 seems to be passing *in front of* the plane of oscillation, directly over the point P'_N located in the vertical median plane towards which the eyes are converged. Also represented is the situation when the bob (now designated by P_{RL}) is moving from right to left and seems to be passing behind the plane of oscillation, directly above the point P'_F located in the vertical median plane towards which the eyes are now supposed to be converged. The filter, as indicated, is retained before the right eye.

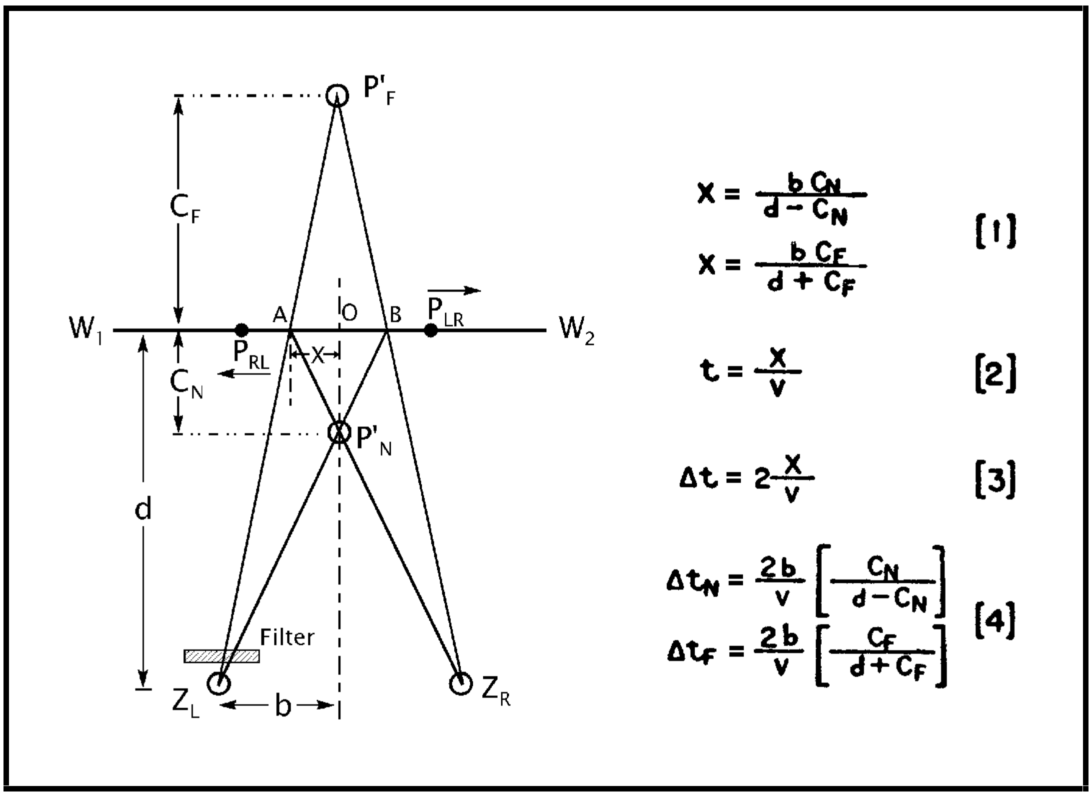

Fig. 1. Schematic representation of the Pulfrich stereophenomenon indicating the stereoscopic space-image in the horizontal plane of fixation.

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When the bob is moving from left to right and appears to be passing directly above the fixation point P'_N, at the near position, then in accordance with the laws of space-projection, the excitation of the left eye was initiated when the bob (now at P_{LR}) was at the point B. Synchronous excitation for the right eye was initiated when the bob was at the point A, a bit farther behind in its path. The time taken for the bob to move from B to P_{LR} represents a theoretical absolute latency for the left eye, and the time taken for the bob to move from A to P_{LR}

represents the slightly longer absolute latency for the right eye with filter. The latency difference (Δt_N) between the two eyes, for the near position, can then be expressed as the time taken for the bob to move from A to B.

By the same reasoning, when the bob is moving from right to left and appears to be passing directly above the fixation point P'_F, at the far position, the time taken for the bob to move from B to A represents the latency difference (Δt_F) between the two eyes. If the velocity of the moving bob is symmetrical on both sides of the vertical median plane, then $\Delta t_N = \Delta t_F$.

From Figure 1 it is evident that for the apparent positions of the bob at P'_N and P'_F respectively,

$$X = bC_N / (d - C_N) \text{ and } X = bC_F / (d + C_F) \qquad [1]$$

where X = 1/2 the distance from A to B; b = 1/2 the distance between the centers of rotation of the two eyes; d = observation distance measured in the vertical median plane from the line joining the centers of rotation; and C_N and C_F represent the distances OP'_N and OP'_F, respectively.[1]

If the velocity of the moving bob is known, it is possible to calculate the latency difference which results from any difference in retinal brightness between the two eyes by experimentally determining the distances OP'N (CN) or OP'F (CF) .

Consider the case in which a vertical rod is made to oscillate with constant velocity (V) in a frontal plane, its path denoted by W_1W_2 (Figure 1).[2] The time (t) required for the rod to move through the distance AO (X) is given by the formula:

$$t = X/V. \qquad [2]$$

The time taken for the rod to move from A to B (or B to A), which represents the latency difference between the two eyes, can then be calculated from the formula:

$$\Delta t = 2X/V. \qquad [3]$$

Substituting for X the relations obtained in Equation [1] we obtain, finally,

$$\Delta t_N = (2b/V)[C_N / (d - C_N)], \text{ and } \Delta t_F = (2b/V)[C_F / (d + C_F)]. \qquad [4]$$

The distances C_N and C_F are hereafter referred to as the near and far displacement of the oscillating target, respectively, and the corresponding latency differences Δt_N and Δt_F, as calculated from Equation [4], are referred to as the near and far latency difference between the two eyes. The greater the displacement, the greater is the stereoeffect.

From purely geometric considerations it is to be noted that, for any constant intensity difference produced between the two eyes, an increase in the observation distance results, theoretically, in an increased stereoeffect, if the corresponding calculated latency differences are to remain constant in magnitude. Similarly, an increase in the velocity of the oscillating target at any given observation distance results in an increased stereoeffect for a constant calculated latency difference. It is to be further noted that, under any given set of conditions, the displacement is theoretically greater when the rod appears to travel behind its plane of oscillation than when in front of its plane of oscillation ($C_F > C_N$). The difference between the two displacements becomes theoretically more marked as the observation distance is increased, and also as X is made larger by increasing the difference in intensity between the two eyes or increasing the velocity of the oscillating rod.

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[1] If the observation distance, d, is measured in the vertical median plane from a reference line other than the one joining the centers of rotation of the eyes (e.g., a line joining (1) the centers of the artificial pupils; (2) the corneal vertices; (3) the principal points of the eyes; (4) the nodal points of the eyes) Equation [1] is still applicable, provided b is now taken to represent 1/2 the distance between the new points of reference (i.e., 1/2 the distance between (1) the centers of the artificial pupils: (2) the corneal vertices; (3) the principal points; (4) the nodal points). The small error which may be introduced by displacements of the selected reference points from the visual axes is neglected in this treatment.

[2] See Lit *(1)* for a similar analysis in the case of a target made to oscillate in a frontal plane with simple harmonic motion.

PREVIOUS EXPERIMENTS

The Pulfrich stereophenomenon has been investigated from many aspects, and the extensive literature summarized by Lit *(1)*. With respect to photometry, duplicity theory and clinical applications, a brief summary and evaluation of the obtained results follows:*

(1) Heterochromatic photometry. Pulfrich employed the stereophenomenon as a new method of heterochromatic photometry. With a colored filter placed before one of the eyes, the colored illumination was then matched with illumination obtained through a graded series of neutral or tinted filters placed before the other eye. Intensity balance for the two eyes was said to exist when the oscillating bob again appeared to move in its true plane of oscillation. The limitations of this method were first established by Von Kries[3] and his co-workers, Engelking and Poos.[4] Heterochromatic matches were consistent with those obtained by other photometric methods only when the observations were made under conditions of high illumination.

(2) Duplicity theory. Von Kries and Engelking and Poos reported peculiar effects when observations were made with one eye dark-adapted and the other eye light-adapted. Although the image seen by the dark-adapted eye appeared brighter, the direction of rotation of the pendulum indicated a greater latency for that eye. Also, despite the shift in maximal sensitivity towards the blue end of the spectrum following dark adaptation (Purkinje phenomenon), the magnitude of the stereoeffect indicated a smaller visual latency when a red rather than a blue filter was placed before the dark-adapted eye. These effects were attributed to a latency differential in the perceptions mediated by the rod and cone mechanisms of the retina, the latency being presumed greater for the rod mechanism.

(3) Clinical applications. The stereophenomenon has been used to test individuals possessing defective color vision. The responses to different colored lights, obtained during the course of dark-adaptation, were correlated with type of color vision deficiency.[5] Liang and Piéron[6] have obtained data which indicate the usefulness of the stereophenomenon as a method for testing color-vision theory.

The application of the stereophenomenon in clinical practice for the early detection of ocular pathology was attempted by Sachs,[7] who found it an extremely sensitive method. With equal illumination presented to each eye, a stereoeffect may be elicited as a result of any optical or ocular condition which produces a difference in the intensity of the retinal images (differentially tinted lenses, anisocoria, corneal or lenticular opacities, etc.). Furthermore, under conditions of equal retinal illumination, the stereophenomenon may also be expected to occur as a result of any visual or systemic pathology which produces a unilateral delay in the transmission of nerve impulses from the retina to the cerebral cortex (retinal pathology, brain tumors, etc.).

Experiments have also been reported on the relationship between the stereophenomenon and stereoscopic acuity.[8]

A PROGRAM OF RESEARCH

Despite the considerable work done, a systematic investigation of the variables which influence the magnitude of the stereophenomenon has not been undertaken. The results reported by previous investigators have shown considerable variability, but these results, obtained under different stimulus

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* To avoid duplication of the extensive bibliography, mention will be made only of the footnote in Lit's paper where the complete references may be obtained.

[3] Lit *(1)*: Footnote 9.

[4] Lit *(1)*: Footnote 10.

[5] Lit *(1)*: Footnote 13.

[6] Lit *(1)*: Footnote 17.

[7] Lit *(1)*: Footnote 15; also see footnote 14.

[8] Lit *(1)*: Footnotes 12, 16.

conditions, cannot be directly compared until the inter-relationships among the stimulus variables have been established. Before attempting to test for individual differences in response, the optimal conditions for testing the stereophenomenon experimentally and applying it clinically will have to be determined. To this end, a program of research designed to obtain complete stimulus specification is required.

Listed below are some of the basic variables which may be expected to influence the magnitude of the stereoeffect and require parametric analysis.

I. The intensity difference in the illumination presented to each of the eyes.

II. The basic level of illumination at which the given difference in retinal illumination occurs.

III. The distance of the plane of oscillation from the eyes.

IV. The velocity of the oscillating target.

V. The width, form and extent of travel of the oscillating target.

VI. The spectral composition of the illumination.

VII. The brightness-contrast between figure and field.

The parametric investigations specified in I and II provide data relating to the theory of space perception, the theory of intensity discrimination and the relationship between the hypothesized *absolute* visual latent period and intensity of illumination.

The independent variables specified in III (distance of observation) and IV (velocity of oscillating target) are essentially ”geometric” in that their influence upon the magnitude of the stereoeffect is predictable from the geometric analysis of the theory of the Pulfrich stereophenomenon discussed in an earlier section of this paper. Any significant departure from predicted effects obtained in III may require analysis in terms of the laws of space perception; departure from predicted effects in IV may provide data for a theory of retinal interaction. Additional data more directly related to retinal interaction effects may be provided by the results obtained from the studies indicated in V (width, form and travel of target) and VII (brightness-contrast).

The results obtained from VI (spectral composition of illumination) will bear most directly on theories concerned with color vision.

In addition to providing appropriate data for theories concerned with some of the most basic visual processes, the results of the proposed research program should also offer many useful clinical and industrial applications.

The study of the influence of binocular differences of intensity produced at various levels of illumination (I and II) has already been undertaken and reported by Lit *(1)*. His results indicate that the magnitude of the stereoeffect depends not only upon the intensity difference produced between the two eyes, but also upon the basic level of illumination at which the intensity difference exists. From his data and interpretation, it may be possible to determine an empirical equation which describes the relationship existing between the hypothesized absolute visual latent period and level of illumination. To avoid deriving an equation of limited significance (limited by the unique values selected for the observation distance, target velocity, target size, etc.) the calculation has been deferred pending investigation of the influence of the stimulus variables requiring parametric analysis. It is the purpose of the present study to investigate III, the effect of the distance of observation upon the magnitude of the stereoeffect.

APPARATUS AND PROCEDURE*

Apparatus: The apparatus used is schematically presented in Figure 2A. The subject is seated in a dark room (D), and binocularly observes the fixation target (FT) and oscillating target (OT) through a pair of circular artificial pupils (E), 2.5 mm. in diameter and adjustable for interpupillary separation. The artificial pupils are attached to eye-tubes which are mounted on the dark room inner-wall. In front of each eye-tube, a filter box (F) is mounted to the outer-wall surface so that by use of appropriate filters the experimenter may control the illumination presented to each eye. The subject's head is kept immobilized by chin and forehead rests.

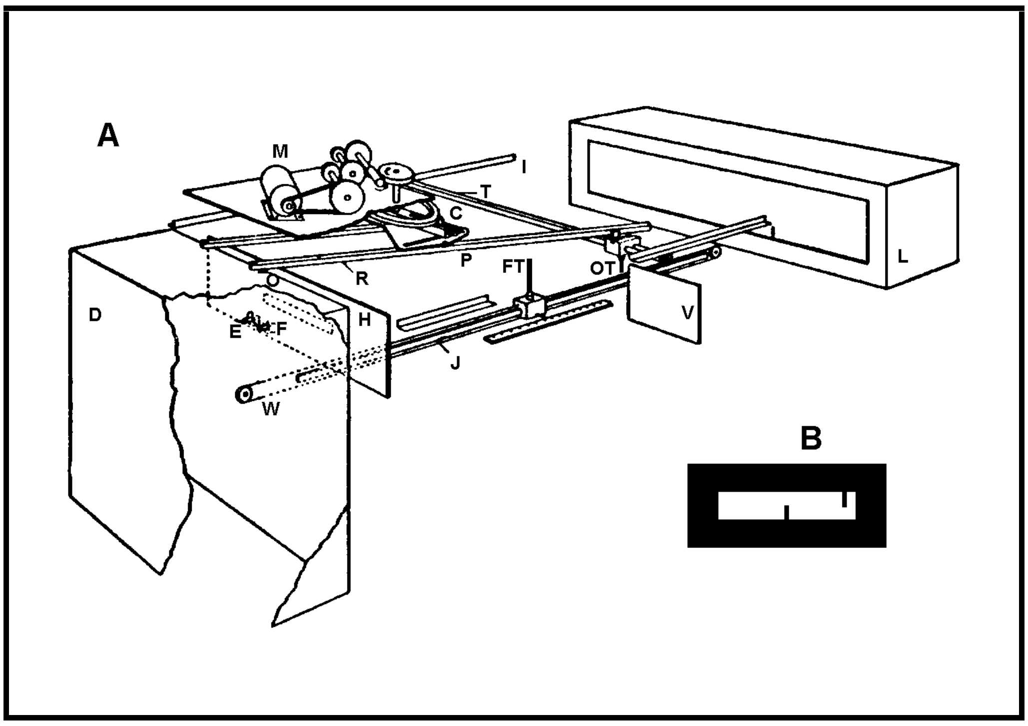

Fig. 2A. Schematic Representation of the Apparatus. The subject is seated in a dark room (D) and binocularly observes the fixation target (FT) and the oscillating target (OT) through a pair of artificial pupils (E). The movement of the oscillating target is restricted to reciprocating constant motion, in a frontal plane, with angular velocity at the subject's eyes identical for each distance of observation. The fixation target may be moved in the vertical median plane either towards or away from the subject by means of the pulley-wheel (W) located in the dark room. The position of the fixation target is read from a millimeter scale. Background illumination is provided by a light box (L). Filters placed in the filter boxes (F) control the illumination presented to each eye. The field of view in the horizontal extent is regulated by a pair of screens (V) placed in the plane of the oscillating target; the vertical extent is determined by the horizontal slit in screen (H). The angular extent of the rectangular field of view

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* The writers wish to express appreciation to their colleague, Professor Edwin W. Bechtold, for his many helpful suggestions in the design of the apparatus. The apparatus was constructed by Mr. Lloyd K. Dutton and Mr. James L. Woods of the Pupin Laboratories, Columbia University.

remains constant for all observation distances.

Fig. 2B. View of Targets as Seen by the Subject. Both targets are seen against the uniform white background produced by the light box. The lower rod is the fixation target located in the subject's vertical median plane. The upper rod is the oscillating target shown at a displaced position in its frontal plane movement. Both angular extent of field and angular size of targets are maintained constant for all distances of observation.

The apparatus consists of three major components: (a) the oscillating target, (b) the fixation target, and (c) the lighting and screening units.

(a) Oscillating target. The oscillating target and its drive mechanism are mounted in a braced angle-iron framework. The oscillating target (OT) is a blackened steel rod rigidly suspended, vertically downward to eye level, from a Jacob's chuck in which it is retained. The chuck is centrally mounted on the undersurface of a supporting carriage which rides on horizontal tracks (T). The tracks can be set in a frontal plane at any predetermined distance from the subject's eyes and clamped to supporting angle-iron rails (I). A slot is cut in the upper surface of the carriage, directly above the Jacob's chuck and perpendicular to the tracks. The carriage receives its movement from a horizontally oscillating drive-rod (R) which is pivoted at position O, a point located in the subject's vertical median plane directly above the line joining the principal points of the eyes. A pin mounted on the undersurface of the drive-rod engages the slot cut in the upper surface of the carriage. The position of the pin from the pivot point (0) is adjustable. Power for the drive-rod is provided by a cam-regulated mechanism (C) which converts constant angular velocity into reciprocating linear velocity, with the central 90 per cent of stroke at constant speed. The power to the drive-rod is applied at a vertical pivot pin (P), permanently mounted on the drive-rod, 75 cm. from 0. An electrically driven gear train (M), containing change gears for control of output r.p.m. in step intervals, provides the cam mechanism with the required angular velocity. Constancy of input r.p.m. for the gear train is maintained to within a 1 per cent error by rheostat regulation of the motor current; constancy is checked with a stroboscopic device.

The oscillating carriage and its drive mechanism, designed to convert constant angular velocity into oscillating linear velocity of constant magnitude in a frontal plane, is shown in plan view in Figure 3. The oscillating target is not indicated. It is suspended from the undersurface of the oscillating carriage (OC) which rides on track (T), clamped on angle-iron rails (I). The carriage is moved by the drive-rod (R), pivoted at O. When the gear train is in motion, the fork-shaped arm (A) rotates with constant

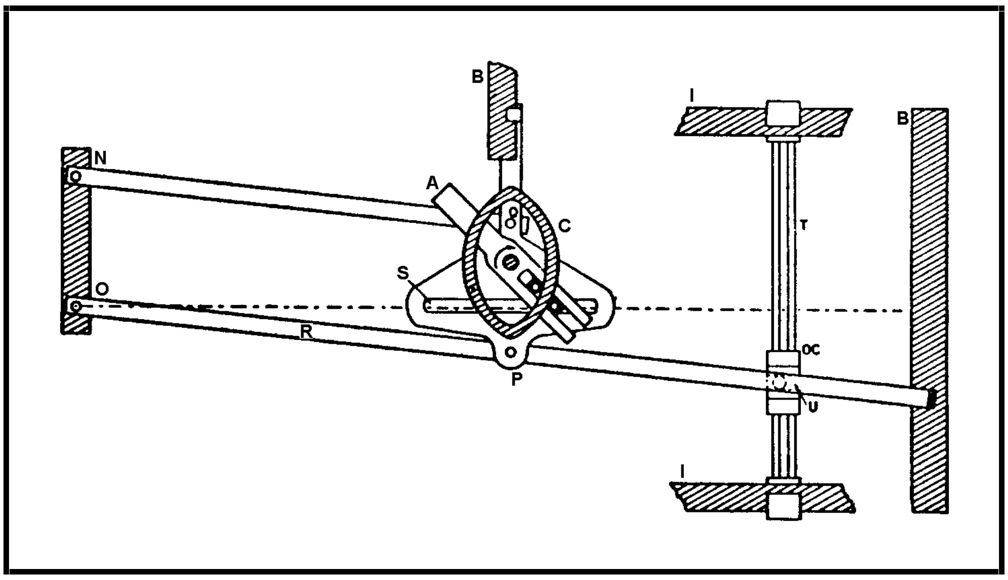

Fig. 3. Mechanism for Providing Horizontal Reciprocating Motion in a Frontal Plane. Speed of OC is constant for the central 90 per cent of stroke and its angular velocity, measured at the subject's eyes [located below (O)], is made identical for all distances of observation.

angular velocity in a direction indicated by the arrow. A follower-carriage which rides in the slotted portion of the arm mounts two rollers, a fixed distance apart, on its upper surface. The position of the carriage from the center of rotation of the arm is determined by a ring-shaped cam (C) on whose inner and outer surfaces the rollers ride. The inner surface of the cam is designed to complement its outer surface so that the follower-carriage, through its rollers, receives positive drive at all positions of the rotating arm. The left and right halves of the cam are symmetrical about the Y axis (defined as the line contained in the frontal plane passing through the center of rotation of the arm). The path traced by the center of the outer roller for the right half of the cam is described by the equation,*

$$\rho = \mathrm{a}\,\theta / \sin\theta \qquad [5]$$

where ρ = radial distance of the outer roller from the center of rotation of the arm; θ = angular distance of the outer roller from the X axis; and a = a constant (distance between center of rotation of the arm and the center of the outer roller when θ equals zero). When the rate of change of θ is constant, the velocity of the frontal plane component (dy/dt) of the outer roller becomes:

$$\mathrm{dy}/\mathrm{dt} = 2\pi \mathrm{a}/\mathrm{T} \ = \text{a constant} \qquad [6]$$

where $2\pi/\mathrm{T}$ equals $\mathrm{d}\theta/\mathrm{dt}$ and T represents the time required for one complete revolution of the arm (A).

The path of the outer roller, described by Equation [5], is also traced by a pin, mounted below the outer roller and projecting from the undersurface of the follower-carriage. The pin engages a slot (S) which is cut perpendicular to the line of bearings of a link pivoted at P and Q. Link PQ is a member of a parallelogram linkage system pivoted at N, O, P and Q. Because PQ remains parallel to a link (ON) permanently fixed in a frontal plane, the slot (S) in PQ is always oriented perpendicular to a frontal plane. Therefore, the frontal plane velocity component of link PQ is identical with that of the pin projecting from the follower-carriage. This velocity component is in turn transmitted to the drive-rod (R) through pivot P. Thus, any point on the drive-rod moves on a circular arc and with a velocity component of constant magnitude in a frontal plane. The drive-rod, through a pin clamped on its undersurface (located at the same distance from O as the plane of the oscillating target), engages the oscillating carriage at slot U. The slot is oriented perpendicular to a frontal plane so that the oscillating carriage responds only to the frontal plane component of the drive-rod pin.

The apparatus may be modified to deliver simple harmonic motion to the oscillating carriage. This is accomplished by removing the cam and follower-carriage mechanism and mounting the pin, which engages slot S, in a fixed position on the rotating arm (A). In this case the equation representing the path travelled by the pin is,

$$\rho = \mathrm{c} \qquad [7]$$

where c = a constant (radial distance of the pin from the center of rotation of the arm). The velocity of the frontal plane component then becomes,

$$\mathrm{dy}/\mathrm{dt} = \left(2\pi \mathrm{c}/\mathrm{T}\right)\cos\theta\,. \qquad [8]$$

This modification does not alter the condition of identical angular velocity of oscillating target (measured at the point 0) for all distances of the plane of oscillation.

In the present experiment, the oscillating target is free to execute constant linear motion in a frontal plane at any specified distance from the subject.[9] Although the magnitude of the angular velocity of the oscillating target remains identical for all target distances, its linear magnitude varies with distance from the subject. Six positions of the oscillating target were selected. The observation distances (= d) corresponding to each of the target positions are: 30 cm., 60 cm., 80 cm., 100 cm., 120 cm. and 150 cm.

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* The cusps which result from the intersection of the symmetrical equations describing the right and left paths of the outer roller have been rounded for mechanical reasons.

[9] Hereafter, specification of distances ("measured from the subject") or of angular magnitudes ("subtended at the subject's eyes") will be made in reference to the point of intersection of the subject's vertical median plane and the line joining the principal points of the eyes. The position of each principal point is taken to lie inside the eye at a distance of 1.3 mm. from the corneal vertex.

The linear velocities of the oscillating target (= V), expressed in centimeters per second, for each of the distances of observation are: 9.91, 19.81, 26.41, 33.02, 39.62 and 49.52, respectively; measured at the subject's eyes each of these linear velocities is equivalent to an angular velocity of 18.91^{o} per sec. The angular extent of stroke of the oscillating target is maintained at 20.5^{o} of visual angle for all observation distances. The angular size of the oscillating target is maintained constant (8.7 minutes of arc) by employing rods whose diameters at each of the testing positions measure 0.75 mm., 1.50 mm., 2.00 mm., 2.50 mm., 3.00 mm. and 3.75 mm., respectively.

(b) Fixation target. The fixation target (FT) is of the same form, color and size as the oscillating rod selected for the given observation distance. It is held upright in a Jacob's chuck which is mounted on the upper surface of a supporting carriage. The carriage rides on a horizontal metal track (J) located in the subject's vertical median plane. The height of the upper end of the fixation target is set on a level with the subject's eyes; when the fixation target is directly below the oscillating target, the targets appear contiguous.

The supporting carriage is made part of an endless-cable and pulley system. By means of a pulley-wheel (W) located in the dark room, the subject may adjust the position of the fixation target, in the vertical median plane, in a direction either towards or away from his eyes. The position of the fixation target is read from a two-meter scale (calibrated in millimeters) which is in contact with a vernier index mounted on the supporting carriage. Free from observable backlash, the mechanism permits the experimenter to estimate the position of the fixation target to within 0.1 mm.

(c) Lighting and screening units. Uniform background illumination is provided by ten 100-watt frosted lamps appropriately mounted in an asbestos-lined, galvanized-iron light box (L) located in a frontal plane 250 centimeters from the subject's eyes. Lamp voltage is maintained constant at 123 volts A.C. by rheostat control. The illuminated surface is a white matte screen attached to the inner rear-wall of the light box. The screen is visible to the subject through a horizontal aperture (165 cm. x 20 cm.) cut out of the front wall of the light box. The brightness of the screen is 610 millilamberts as calculated from measurements with a Macbeth illuminometer. For an artificial pupil 2.5 mm. in diameter, this is equivalent to a retinal brightness of 9510 photons.[10] Differences in retinal brightness between the two eyes are obtained by placing appropriate neutral Wratten filters in the filter boxes (F) provided.

The screening unit is designed to present the subject with a rectangular field of view whose angular dimensions are identical for all distances of the oscillating target. A constant angular extent of field of 4.2^{o} in the vertical direction is obtained by a horizontal slit (26 cm. x 1.5 cm.) cut, at eye level, in a black screen (H) located 21 cm. in front of the subject. A constant angular extent of field of 21.6^{o} in the horizontal direction is obtained by means of a pair of vertical screens (V) [only one is shown in Figure 2A] adjusted symmetrically in the plane of the oscillating target, 0.5 beyond the end-points of its reciprocating stroke. Placing the vertical screens in the plane of oscillation serves to minimize physiological diplopia; placing the horizontal slit close to the subject provides for complete freedom of movement of the fixation target at all target stations. As shown in Figure 2B, the subject sees the upper end of the fixation target and the lower end of the oscillating target contained within a constant angular field of view, for all distances of observation, against the uniform white background produced by the light box.

Procedure: A well trained, emmetropic subject (A.H.) with normal visual acuity and good stereopsis was used. Before testing at each of the six observation distances (d = 30 cm., 60 cm., 80 cm., 100 cm., 120 cm. and 150 cm.), the separation between centers of the artificial pupils was adjusted to conform to the changes in convergence position of the eyes; the inter-principal point separation of the eyes at each of the observation distances was calculated to be, respectively, 64.8 mm., 65.9 mm., 66.2 mm., 66.3 mm., 66.5 mm. and 66.6 mm. (= 2b in accordance with footnote [1], above). At each distance of observation, the subject made settings under conditions which included seven differences in retinal brightness between the two eyes. The retinal illumination

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[10] Photons = $(10/\pi)$ x pupil area in square millimeters x brightness in millilamberts.

of the right eye, I_R, was kept constant at 300 photons (log I_R = 2.478) by retaining a filter of optical density l.50 (= 3.16% transmission) before the eye. The retinal illumination of the left eye, I_L, was increased in discrete steps by employing filters of optical density 1.40, 1.30, 1.00, 0.70, 0.50, 0.20 and 0.00. For the above stated conditions of retinal illumination the calculated values of log (I_L/I_R) are: 0.1, 0.2, 0.5, 0.8, 1.0, 1.3 and 1.5 respectively. The order of testing at the six distances of observation was randomized and testing at any one of the distances of observation constituted a session.

In any given experimental session, a 30-minute period of dark adaptation preceded the observations. With filters of equal optical density (1.50) placed before the eyes ($I_L = I_R$ = 300 photons) the oscillating target was held stationary and laterally displaced 5.73^{o} to the right or left of the subject's vertical median plane. The subject was required, at the given distance of observation, to position the fixation target with respect to the displaced oscillating target until both appeared to lie in a frontal plane. Five such determinations of frontal plane alignment were made at each of the two lateral positions of the oscillating target to obtain a measure of the subject's stereoscopic sensitivity at each of the six distances of observation. In addition. under conditions of equal retinal illumination ($I_L = I_R$ = 300 photons), the oscillating target was set into motion, and the subject was required to adjust the fixation target directly below what appeared to be the frontal plane of oscillation. Ten settings were made at each distance of observation to obtain a measure of the localization error of the oscillating target when in motion [Lit *(1)*, pp. 179-180.]. Following these preliminary settings, the basic measurements involved in the experiment were made. Under each of the seven conditions of retinal brightness differences, the subject was required to locate the oscillating target in both its near and far apparent positions, first when the fixation rod was moved away, and again when it was moved towards him. A total of six readings was made at each of the two positions for each of the given retinal brightness differences. A 3-minute period of light adaptation followed each increase in retinal brightness.

RESULTS AND DISCUSSION

Table I summarizes the experimental data obtained for subject A. H. at each of the six distances of observation (d). The seven differences in retinal illumination produced between the two eyes at each of the distances of observation are expressed as log (I_L/I_R), where I_L and I_R represent the retinal illumination of the left and right eye, respectively. The indicated values of log (I_L/I_R) are obtained by subtracting the optical density of each of the filters placed before the left eye (D_L) from the optical density of the filter retained before the right eye (D_R) [i.e., $D_R - D_L = \log I_L - \log I_R = \log (I_L/I_R)$]. The apparent displacements of the oscillating target, measured from the plane of oscillation, are given (in centimeters) for both the near and far positions (C_N and C_F); each entry is the mean of six readings. The corresponding near and far latency differences (Δt_N and Δt_F), calculated from Equation [4], and their averaged values ($\overline{\Delta t}$) are all expressed in milliseconds.[11]

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[11] The required six values of 2b/V which were substituted in Equation [4] to calculate the magnitude of the latency differences (Δt_N and Δt_F) at each of the respective distances of observation are 0.655, 0.333, 0.251, 0.201, 0.168 and 0.134. These values are based upon the previously stated magnitudes of the linear velocity (V) and the corresponding distance between the principal points of the eyes (2b), existing at each of the distances of observation (d).

TABLE I

DISPLACEMENTS AND CALCULATED LATENCY DIFFERENCES PRODUCED BY DIFFERENCES IN RETINAL BRIGHTNESS AT SIX DISTANCES OF OBSERVATION

C_N and C_F* refer, respectively, to the near and far apparent displacements; each entry is based on the mean of six readings. Δt_N and Δt_F* are the corresponding near and far latency differences calculated from Equation [4]; the averaged values of Δt_N and Δt_F are given by $\overline{\Delta t}$. The retinal brightness of the right eye is kept constant at 300 photons (log I_R = 2.478).

Observation Distance	Log I_L/I_R	C_N (cm.)	C_F (cm.)	Δt_N (msec.)	Δt_F (msec.)	$\overline{\Delta t}$ (msec.)
30 cm.	0.10	0.135	0.058	2.96	1.26	2.11
	0.20	0.148	0.068	3.25	1.48	2.37
	0.50	0.290	0.197	6.39	4.27	5.33
	0.80	0.395	0.273	8.73	5.90	7.32
	1.00	0.488	0.302	10.83	6.52	8.68
	1.30	0.542	0.410	12.05	8.82	10.44
	1.50	0.627	0.422	13.98	9.08	11.53
60 cm.	0.10	0.533	–0.150	2.98	–0.83	1.08
	0.20	0.962	–0.130	5.42	–0.72	2.35
	0.50	1.188	0.542	6.72	2.98	4.85
	0.80	1.545	0.907	8.79	4.95	6.87
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	1.30	2.222	1.420	12.80	7.69	10.25
	1.50	2.448	1.622	14.15	8.76	11.46
80 cm.	0.10	1.222	0.112	3.89	0.35	2.12
	0.20	1.333	0.053	4.24	0.17	2.21
	0.50	2.682	0.720	8.69	2.23	5.46
	0.80	3.057	1.332	9.95	4.10	7.03
	1.00	3.488	1.720	11.42	5.27	8.35
	1.30	3.773	2.017	12.40	6.16	9.28
	1.50	3.833	2.030	12.61	6.20	9.41
100 cm.	0.10	2.017	–0.130	4.14	–0.26	1.94
	0.20	2.413	0.167	4.97	0.33	2.65
	0.50	3.352	0.913	6.97	1.82	4.39
	0.80	4.327	2.077	9.09	4.09	6.59
	1.00	5.063	2.747	10.71	5.37	8.04
	1.30	5.902	3.185	12.60	6.20	9.40
	1.50	6.170	3.452	13.21	6.70	9.96
120 cm.	0.10	2.340	–0.527	3.34	–0.74	1.30
	0.20	2.797	–0.270	4.00	–0.38	1.81
	0.50	4.970	1.897	7.25	2.61	4.93
	0.80	6.233	2.845	9.19	3.88	6.54
	1.00	7.008	3.253	10.40	4.43	7.42
	1.30	8.498	5.290	12.78	7.08	9.93
	1.50	8.933	4.277	13.49	5.77	9.63
150 cm.	0.10	3.528	–1.977	3.24	–1.80	0.72
	0.20	4.232	–1.863	3.90	–1.69	1.11
	0.50	7.367	2.528	6. 94	2.23	4.59
	0.80	9.765	3.635	9.36	3.18	6.27
	1.00	11.527	5.875	11.19	5.07	8.13
	1.30	13.227	6.632	13.00	5.69	9.35
	1.50	14.715	8.880	14.62	7.51	11.07

* The negative sign for values of C_F and Δt_F are used to indicate cases where the far position of the oscillating target lies closer to the subject than does the plane of oscillation.

LOCALIZATION ERROR. It is to be noted in Table I that several entries for C_F (and their corresponding Δt_F values) are preceded by a minus sign. The negative sign is used in all cases where the apparent far position of the oscillating target lies closer to the subject than does the true plane of oscillation. Such cases reflect the existence of a localization error. With filters of equal optical density (1.50) placed before each eye, the oscillating target appeared to move in a frontal plane which was localized by the subject, at each of the distances of observation, in a plane nearer than the plane defined by the true distance of the oscillating target. Consequently, when small differences in retinal illumination were produced between the two eyes, the resulting stereoeffects were of insufficient magnitude to locate the apparent far position of the oscillating target beyond the plane of oscillation. For A. H., the magnitude of the localization error (in centimeters) increases as the distance of the plane of oscillation increases. At the six observation distances used in the experiment, the amounts by which the true plane of oscillation appeared displaced in a direction towards the subject were: 0.016 cm., 0.350 cm., 0.648 cm., 0.913 cm., 1.259 cm., and 2.579 cm., respectively. [These displacements are equivalent to latency differences (obtained by substitution in Equation [4] and solving for Δt_N) of 0.35 msec., 1.95 msec., 2.05 msec., 1.85 msec., 1.78 msec., and 2.05 msec.; the displacements correspond to stereoscopic difference-angles of 24 sec., 132 sec., 139 sec., 125 sec., 121 sec., and 139 sec.].

The reason for the localization error has not been established. In the previous experiment reported by Lit, in which A. H. acted as one of the subjects, a localization error of comparable magnitude and direction was obtained. A separate investigation is being planned to determine the stimulus factors which influence its magnitude and direction. Fortunately, the localization error for A. H. is of sufficiently small magnitude as not to obscure the relationships under major consideration in the present investigation.

The data given in Table I are graphically presented in Figures 4 to 7. The curves drawn through the points have been fitted by visual inspection.

(1) Displacement. Figures 4 and 5 show the respective effects of (a) retinal illumination, log (I_L/I_R) and (b) observation distance, d, upon the magnitude of the near and far displacements (C_N and C_F).

(a) Displacement as a function of log (I_L/I_R). Figure 4 shows the relationship

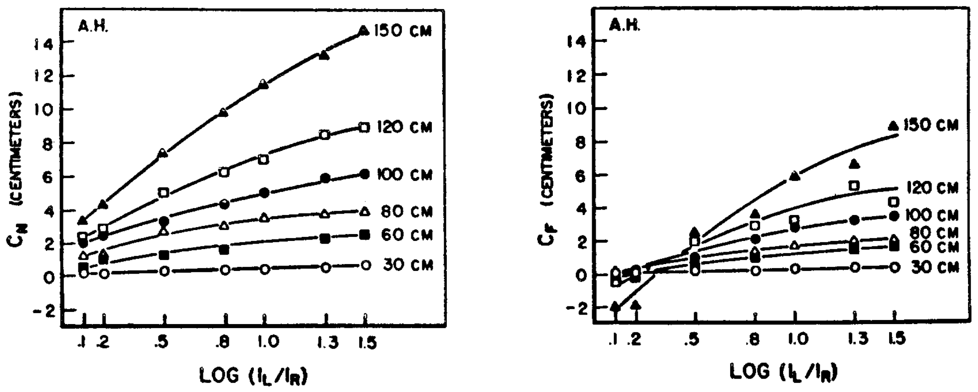

Fig. 4. Displacement as a Function of log (I_L/I_R). The near and far displacements (C_N and C_F) produced by the respective values of log (I_L/I_R) are given for each of the distances of observation indicated by the number accompanying the curve. Each point is the mean of six readings, and the curves drawn through the points have been fitted by visual inspection. Negative values of C_F indicate that the apparent far position of the oscillating target lies closer to the subject than does the true plane of oscillation, an effect which is produced by a localization error which is present when the oscillating target is observed under conditions of equal retinal illumination.

obtained between near and far displacements (C_N and C_F) and log (I_L/I_R). The number accompanying each curve represents the distance of observation, d, expressed in centimeters.

The relationship existing between displacement and log (I_L/I_R) may be characterized by the following general observations: (a) both C_N and C_F increase as log (I_L/I_R) increases; (b) the rate of increase is not the same for each of the curves; slope increases as distance of observation increases; (c) for a given value of log (I_L/I_R), both C_N and C_F increase progressively as the distance of observation is increased; (d) for a given magnitude of C_N or C_F, the value of log (I_L/I_R) necessary to produce the given displacement becomes progressively smaller as the distance of observation is progressively decreased.

The results described above are qualitatively consistent with the theory and geometric analysis of the Pulfrich stereophenomenon discussed in earlier sections of this paper. However, a minor deviation from theoretical predictions is to be noted. For a given value of log (I_L/I_R), produced at any given distance of observation, the magnitude of C_N is greater than the corresponding value of C_F, particularly for the greater distances of observation. This result duplicates the findings obtained by Lit in the previous experiment with A. H. acting as subject. Additional study is required to account for the persistent relative foreshortening of the far displacement with respect to the near displacement.

(b) Displacement as a function of d. Figure 5 shows the results obtained between near and far displacements C_N and C_F and *distance of observation* (d). The number accompanying each curve represents the value of log (I_L/I_R) holding for that curve. The figure is given to demonstrate more directly (a) that for any given value of log (I_L/I_R), both C_N and C_F increase as the distance of observation increases, and (b) that the rate of increase is not the same for all curves; slope increases as log (I_L/I_R) increases. The above statements are entirely consistent with predictions derived from the geometric analysis of the Pulfrich effect. Slight discrepancies with respect to C_F are to be noted for the two lowest curves of log (I_L/I_R).

(2) Latency Difference. Figures 6 and 7 show the respective effects of (a) retinal illumination, log (I_L/I_R) and (b) observation distance, d, upon the rnagnitude of Δt_N, Δt_F, and $\overline{\Delta t}$. Δt_N and Δt_F were computed from Equation [4] for each of the corresponding values of C_N and C_F; each point on the curves

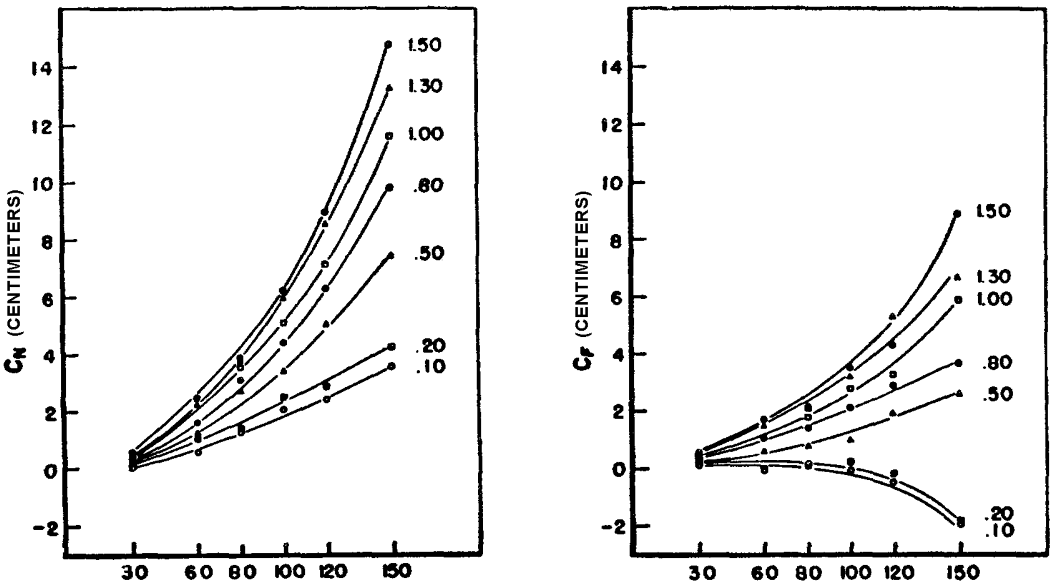

Fig. 5. Displacement as a Function of d. The near and far displacements (C_N and C_F) obtained at each of the distances of observation (d) are given with log (I_L/I_R) as parameter. The number accompanying each curve represents the value of log (I_L/I_R) holding for that curve.

is based upon the mean of 6 readings. The points for $\overline{\Delta t}$ represent the averaged values of Δt_N and Δt_F and are each based on a total of 12 readings.

(a) Latency difference as a function of log (I_L/I_R). Figure 6 shows the relationship between the calculated values of Δt_N, Δt_F and $\overline{\Delta t}$ and log (I_L/I_R) obtained at each of the distances of observation indicated by the number accompanying the curves.

The obtained curves indicate that both Δt_N and Δt_F (as well as their averaged values, $\overline{\Delta t}$) progressively increase as the value of log (I_L/I_R) increases. This result is similar to the relationship obtained between C_N and C_F and log (I_L/I_R). However, for the

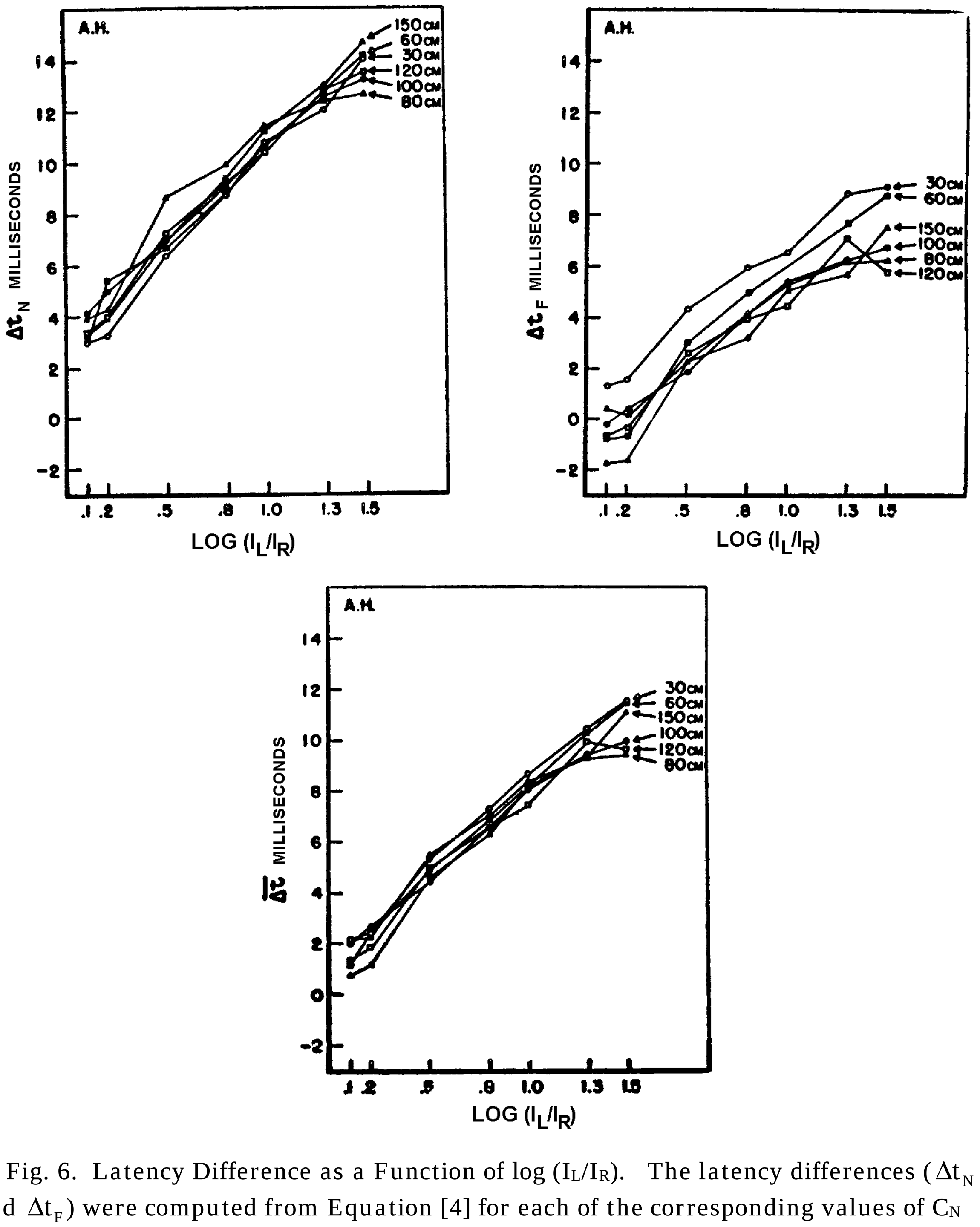

Fig. 6. Latency Difference as a Function of log (I_L/I_R). The latency differences (Δt_N and Δt_F) were computed from Equation [4] for each of the corresponding values of C_N and C_F produced by the respective values of log (I_L/I_R); each point is based upon the mean of six readings. The values of $\overline{\Delta t}$ represent the averaged values of Δt_N and Δt_F. The minus sign is retained when calculating Δt_F for negative values of C_F.

case of either Δt_N, Δt_F or $\overline{\Delta t}$, the rate of increase with log (IL/IR) appears to be the same for all distances of observation. Furthermore, with the exception of two curves at Δt_F (at the 30 cm. and 60 cm. testing distance) there is no evidence of a tendency for the curves to separate according to the respective value of the distance of observation. The curves representing values of $\overline{\Delta t}$ show considerable overlapping and are all confined within a narrow band whose range does not exceed two milliseconds for all but one of the given values of log (IL/IR). These results are in complete *quantitative* agreement with the geometrical analysis of the Pulfrich stereophenomenon.

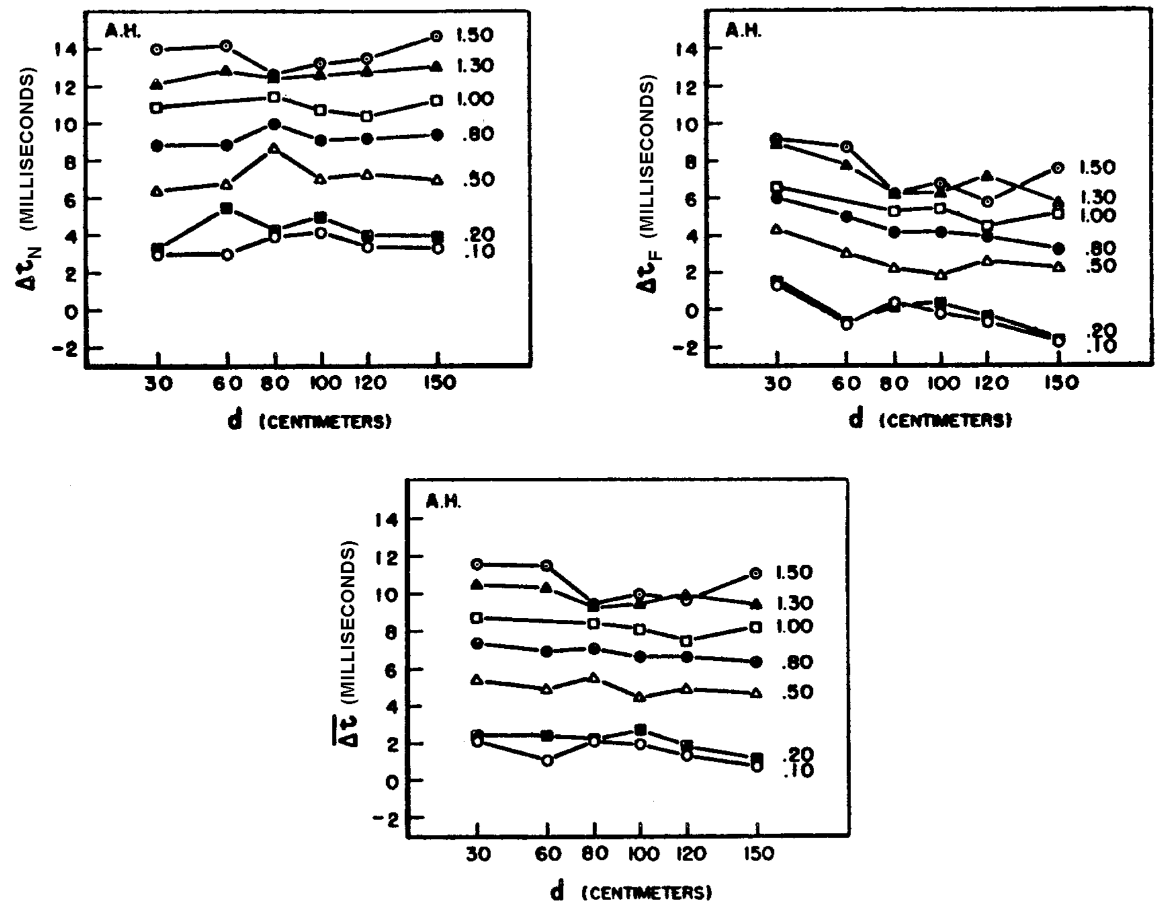

Fig. 7. Latency Difference as a Function of d. The calculated latency differences (Δt_N, Δt_F and $\overline{\Delta t}$) obtained at each of the distances of observation (d) are shown for the respective values of the parameter, log (IL/IR).

(b) Latency difference as a function of d. A more direct demonstration of the independence of distance of observation upon the magnitude of the calculated latency differences may be obtained from Figure 7. The latency differences (Δt_N, Δt_F and $\overline{\Delta t}$) obtained for each of the respective values of log (IL/IR) are given as functions of the distance of observation. It is to be particularly noted that, for any given intensity difference, expressed as log (IL/IR), the calculated values of Δt_N, Δt_F and $\overline{\Delta t}$ remain essentially constant for all observation distances. The tendency of the curves for Δt_F to deviate slightly from a horizontal line may represent variability of response. Further work with additional subjects is required for complete analysis.

In summary, the basic results of the present investigation are consistent with those predicted from the theory and geometric analysis of the Pulfrich stereophenomenon. It is demonstrated in accordance with theory that, for a given difference in retinal brightness, the magnitude of the stereoeffect varies with distance of observation to produce a constant calculated latency difference.

The magnitude of the constant latency difference increases as the intensity difference between the two eyes increases, and at a rate which probably depends upon the relationship which exists between the *absolute* visual latent period and level of illumination. The influence of accommodation, convergence and other factors associated with changes in observation distance do not appear to have any markedly disturbing effects upon settings made under the conditions of the present experiment. The agreement found between theoretical and experimental results serves to confirm the validity of the geometric assumptions underlying the theory of the Pulfrich phenomenon. Latency difference measurements obtained at distances of observation lying within the limits described in the present experiment, and resulting from any given difference in retinal illumination, may be considered replications; the obtained latency differences for the various distances of observation may be combined into a single statistical measure of central tendency with respect to the given difference in retinal illumination. It is anticipated that the minor discrepancies observed between theoretical and experimental results will become clarified as data from additional subjects become available for each of the variables described in the proposed research program.

SUMMARY

(1) The present experiment is concerned with a stereoscopic effect first described and analyzed by Pulfrich: an oscillating target appears to rotate out of its plane of oscillation when binocularly observed under conditions of unequal retinal illumination. The oscillating target appears nearer than it really is for one direction of stroke and farther than it really is for the return stroke. The apparent displacements are attributed to differences in the visual latent periods for the two eyes resulting from the differential retinal illumination. Geometric analysis of the theory of the Pulfrich effect allows for calculation of the corresponding latency differences when the near and far displacements in the vertical median plane are determined experimentally.

(2) The magnitudes of the apparent near and far displacements (C_N and C_F) and the corresponding latency differences (Δt_N and Δt_F), produced by differences in retinal illumination [expressed as log (I_L/I_R)], are measured as a function of the distance of the oscillating target from the subject's eyes. Data are obtained for seven differences in retinal illumination between the two eyes produced at each of six distances of observation (d) ranging from 30 cm. to 150 cm.

(3) The apparatus used to obtain the data is described. It provides for an oscillating target which is free to execute constant linear

motion in a frontal plane at any specified distance from the subject. The magnitude of the linear velocity varies with distance from the subject, but its angular velocity (18.91° per sec.) remains identical for all target distances. The angular extent of stroke of the oscillating target (20.5° of visual angle) and the angular size of the oscillating target (8.7 min. of arc) are maintained constant for all distances of observation. The angular extent of the rectangular field of view (21.6° x 4.2°) remains constant at all observation distances.

(4) The obtained experimental results are consistent with those predicted from the theory and geometric analysis of the Pulfrich stereophenomenon: for any of the given differences in retinal brightness, the magnitude of the stereoeffect varies with distance of observation to produce a constant calculated latency difference. Minor discrepancies between theoretical and experimental results are discussed.

(5) A program of research is proposed, designed to determine the influence of several basic stimulus variables upon the magnitude of the stereoeffect. In addition to providing data for theories concerned with some of the most basic visual processes, the results of the proposed research program should offer many useful clinical and industrial applications.

REFERENCES

1. Lit, A., The Magnitude of the Pulfrich Stereophenomenon as a Function of Binocular Differences of Intensity at Various Levels of Illumination. Amer. J. Psychol., 1949, 62, 159 -181.
2. Pulfrich, C., Die Stereoskopie im Dienste der isochromen und heterochromen Photometrie. Naturwissenschaften, 1922, 10, 553 - 564, 569 - 574, 596 - 601, 714 - 722, 735 -743, 751 - 761.

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